Week 4 - Trivalent Account for Presuppositions & The Definite Descriptor

The - Russell vs. Strawson

The classic Russellian view of the definite descriptor **The** in English is similar to that of quantification (Russell 1905). (1) formalises the sentence 'The cat purred'. The predicate logic captures the existence of at least one cat, the uniqueness condition that there is at most one cat, and the predication.

(1) $\exists x.[\text{Cat}(x) \wedge \forall y(\text{Cat}(y) \rightarrow y = x) \wedge \text{Purred}(x)]$

Crucially, this formula predicts that if there is no cat the sentence is simply false. It's a proposition that is entirely asserted with no presupposition.

Strawson (1950) countered Russells claim by proposing that the definite descriptor actually presupposes existence and uniqueness. Part of his reasoning being that the existence presupposition seems to survive negation:

(2) The Chandler House cat **reddid not** purr.
$\Longrightarrow$ There exists a Chandler House cat.

Using Russell's system, (2) would be judged as true (the affirmative case is judged as false so its negation is true). However, Strawson argues that most people wouldn't take as true, instead they would be likely to say something like 'But wait, there is no Chandler House cat.'

This intuition led Strawson to believe that if there is no Chandler House Cat then something like (2) is neither true nor false. Instead, the sentence feels 'off'. In other words, if a presupposition P is false (i.e., there is no cat), then the sentence presupposing P cannot even be evaluated as either true or false. One way of capturing this state of 'not-evaluate' is to say that the expression overall is **undefined**, which is denoted as #. We previously referred to this as **presupposition failure**. A related concept is the idea of **truth-gaps**, which occur when classical truth values are non-exhaustive (c.f., vagueness).

Question: Do you have the same intuition as Strawson??

ι Operator

One way of capturing 'the' is by using what is called the **ι-operator**, and by extension, **trivalent logic**. This approach handles the presupposition at different levels of the semantic architecture, and intends to capture and formalise instances of presupposition failure.

The ι (iota) operator is an operator that binds onto a variable to result in an expression of type e such that the entity denotes a unique individual satisfying P, otherwise it denotes #. If the entity is not unique and thus denotes #, this value makes it up the composition and the whole expression is #.

(3) $[\![\text{the}]\!] = \lambda P.\iota x.P(x)$ (From C&C 2025)

(4) $$[\![\iota x.\,\text{cat}(x)]\!]^w = \begin{cases} d & \text{if } \{x \in D : \text{cat}(x)\} = \{d\} \\ \# & \text{otherwise} \end{cases}$$

What this formula is telling us is that the denotation of cat is the unique individual d such that cat(d). If there is no d or more than one d the denotation is simply undefined.

From a compositional level we can take the definite determiner to be of semantic type $\langle\langle e,t\rangle e\rangle$: a function that takes the predicate of type $\langle e,t\rangle$ (e.g., 'cat') and outputs the DP of semantic type e to be combined with something else.

The ι-operator is also used in C&C to formalise possessive presuppositions, with the compositional formula for the possessive morpheme ''s' being as follows:

(5) $\text{'s} \rightarrow \lambda x \lambda P \,.\, \iota y \,.\, P(y) \wedge \text{has}(x,y)$

Given how useful it is to have a kind of unifying operator for uniqueness presuppositions, Beaver and Kramer (2001) also propose a partial operator ∂ Which provide the same definedness conditions that ι does but for other kinds of presupposition determiners. Keep in mind that the original Heim and Kratzer (1998) introduces definedness conditions with the colon-dot notation. Here, the presupposition is written between the colon and dot; the following is an example for 'neither' presupposing the existence of two 'things' from C&C

(6) $\lambda P \lambda Q : |P| = 2 \,.\, \neg\exists x[P(x) \wedge Q(x)]$

Trivalent truth-conditions

Now lets zoom out and look at **Trivalent truth-conditions**. In this system, we take the extension of a sentences to have three possible semantic values: True (1), False (0), and undefined (#).

(7) Jisu forgot the water **the** plants.
→ Presupposes that there exists some plants.

(8) $$[\![(7)]\!]^w = \begin{cases} 1 & \text{if Jisu watered the plants in } w \\ 0 & \text{if Jisu did not water the plants in } w \\ \# & \text{otherwise} \end{cases}$$

Notice that classical truth conditions (1 & 0) still presuppose that there exists a plant! The # value is reserved for cases in which the presupposition fails i.e., there are no plants. We denote # as *otherwise* because truth conditions have to be **exhaustive** of all possibilities.

An advantage of this trivalent system is that it allows us to depict how we expect projection to work, especially under operators. There are several ways to depict these properties based on what kind of system we want to use.

Firstly, both Strong and Weak Kleene stipulate the same thing for negation: negation maps true to false and *vice versa* but preserves undefinedness:

Weak Kleene and Strong Kleene truth tables for $\neg$

	$\neg$
T	F
F	T
#	#

	$\neg$
T	F
F	T
#	#

Weak Kleene truth tables for $\wedge$ and $\vee$

$\wedge$	T	F	#
T	T	F	#
F	F	F	#
#	#	#	#

$\vee$	T	F	#
T	T	T	#
F	T	F	#
#	#	#	#

Strong Kleene truth tables for $\wedge$ and $\vee$

$\wedge$	T	F	#
T	T	F	#
F	F	F	F
#	#	F	#

$\vee$	T	F	#
T	T	T	T
F	T	F	#
#	T	#	#

The difference between these two system lies in exactly what we consider 'undefinedness' to actually be. In Weak Kleene, # literally means **undefined** – It makes sense that if part of an expression is undefined, the entire part will also be undefined.

On the other hand, in Strong Kleene, # represents **uncertainty**. In other words, # means 'maybe 1 or maybe 0'. (9-b) here depicts the conditions for conjunction. The falsity condition states that as long as one conjunct is false the other conjunct doesn't actually matter, so the uncertainty doesn't affect the overall expression being false.

(9) a. $([\![\phi]\!] \wedge [\![\psi]\!] = 1)$ if $[\![\phi]\!] = 1 \wedge [\![\psi]\!] = 1$
  b. $([\![\phi]\!] \wedge [\![\psi]\!] = 0)$ if $[\![\phi]\!] = 0 \vee [\![\psi]\!] = 0$

Question: Let's say you had the sentence: '#[The King of France is bald] and F[Germany is in Asia]. Weak Kleene predicts the entire expression to be #, while Strong Kleene predicts it to be F. What are your intuitions?

So which is better for our intuitions of presupposition projection in natural language? For the most part, people take Strong Kleene to be the default set of operators. There are several reasons for this, including the fact that Weak Kleene may over-generate infelcitiy (Do you agree?)

Another reason is that Strong Kleene allows us to model presupposition filtering:

(10) If France has a king, then the king of France is bald.

Under Weak Kleene, it would predict that the entire expression (10) is infelicitous, however Strong Kleene allows the expression to remain true even if the consequence is undefined. Thats because under Strong Kleene if p = 0 and q = # then (p→q) = 1 while in Weak Kleene (p→q) = #.

Moreover, Strong Kleene gives allows us to model local accommodation:

(11) Either there is no king of France, or the king of France is bald.

In (11), we would take the first conjunct to be true and the second to be #. Weak Kleene would predict the entire expression to be # (if p = 0 and q = # then (¬p ∨ q) = #). On the other hand, Strong Kleene predicts that the overall expression is 1.

Bridging Principle
How does # relate to the overall idea of context that we have been referring to continuously throughout this course? Stalnaker introduces a **Bridging Principle** that connects pragmatic infelicity to our idea of semantic definedness:

> "To say that a speaker presupposes a proposition p in a context C is to say that p is taken for granted in C, that is, that p is entailed by every possible world in C." (Stalnaker 1974)

In other words, if an expression is undefined in any context-world, it cannot be felicitously asserted. It is bridging the pragmatic conditions on the context update with the truth values in specific worlds. What this bridge essentially tells us is when it is possible to make an assertion and when it is not.

Quantified Sentences
Lets look at a sentence ⟦Every NP VP⟧, assuming Strong Kleene. Remember that in predicate logic, universal and existential quantifiers are duals of eachother: $\forall x.\neg\phi$ is equivalent to $\neg\exists x\ .\ \phi$.

(12)

$$[\![\text{NP}]\!] = \lambda x. \begin{cases} 1 \\ 0 \\ \# \end{cases} \qquad\qquad [\![\text{VP}]\!] = \lambda x. \begin{cases} 1 \\ 0 \\ \# \end{cases}$$

(13) $$[\![\text{Every NP VP}]\!] = \begin{cases} 1 & \forall x[[\![\text{NP}]\!](x) = 1 \rightarrow [\![\text{VP}]\!](x) = 1] \\ 0 & \exists x[[\![\text{NP}]\!](x) = 1 \wedge [\![\text{VP}]\!](x) = 0] \\ \# & \text{otherwise} \end{cases}$$

The expression is evaluated as # when the presupposition fails, which means the presupposition is satisfied in both 1 and 0. So, we can rewrite the presupposition as the **disjunct of values 1 and 0 of (13)**.

(14) (13) presupposes that: $\forall x[[\![\text{NP}]\!](x) = 1 \rightarrow [\![\text{VP}]\!](x) = 1] \vee \exists x[[\![\text{NP}]\!](x) = 1 \wedge [\![\text{VP}]\!](x) = 0]$

So lets plug this into the expression 'Every fat man pushed his bike.' This theory would predict the presupposition to be either: Every fat man has a bike and is pushing it **or** at least one fat man has a bike but is not pushing it. We can see that the second presupposition is weaker, it only presupposes that at least one fat man has a bike, and it's not required that all of them have one, which goes against out intuition

What about 'a'? Let's take the sentence 'a linguist forgot to water her plants.'

(15) $$[\![\text{a linguist forgot to water her plants.}]\!] = \begin{cases} 1 & \exists x[[\![\text{linguist}]\!](x) = 1 \rightarrow [\![\text{f.t.w.h.p}]\!](x) = 1] \\ 0 & \forall x[[\![\text{linguist}]\!](x) = 1 \rightarrow [\![\text{f.t.w.h.p}]\!](x) = 0] \\ \# & \text{otherwise} \end{cases}$$

Again, the presupposition is predicted to be the disjunction of 1 and 0 – either there is a linguist who has plants and forgot to water them or every linguist has a plant but is not forgetting to water it. It seems like 'every' has a stronger presupposition than 'a'?

Question: What truth-conditions/presuppositions do you get for the quantifier 'Exactly two'?

Fox's Fix

In Fox (2012), he adopts the Strong Kleene trivalent system described above. He claims that the semantic presupposition is the disjunction as demonstrated above, however the pragmatics dictates that it is a strange thing to presuppose, and thus strengthens the presupposition to a universal one. This mechanism is in line with proposals on how some people choose to deal with the proviso problem.

Fox stipulates that different quantifiers have different 'presupposition strengths' e.g., 'every, exactly one' have strong presuppositions because they 'check all relevant cases for definedness' while 'no, few' have weak presuppositions because they only look at cases where the predicate is true while undefined cases do not necessarily affect the outcome. This projects a weaker presupposition (maybe some students once smoked, but not all in (16)).

(16) No student stopped smoking.

One would note that this goes against Stalnakers bridge because we are essentially allowing # worlds in the context. Fox is cautious of this, choosing not to abandon the bridge but rather to re-contextualise it:

> **Parameterized Bridge Principle (Fox's version):**
> "The presupposition strength of an operator determines the set of individuals for

> which it requires definedness of the predicate it applies to. The stronger the presupposition, the larger this set."

Acknowledgments
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List of References

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Stalnaker, Robert (1974). "Pragmatic presuppositions". In: *Semantics and Philosophy*. Ed. by Milton K. Munitz & Peter Unger, 141–117.